wave; T, temperature; T_0, temperature of the ambient medium; T_m, maximum temperature of the steady state; t, time; Δt_{me}, time from the onset of localization to the melting of the material; c, ρ, k, the heat capacity, the density, and the thermal conductivity of the dielectric; α, heat-transfer coefficient; c_s, the Stefan–Boltzmann constant; z, emissivity of the dielectric; Θ, φ, η, τ, dimensionless temperature, electric-field strength, a coordinate, and time.

LITERATURE CITED

1. Yu. M. Misnik, Fundamentals of Frozen Rock Thawing by Means of SHF Fields [in Russian], Leningrad (1982).
2. A. G. Merzhanov, V. A. Raduchev, and É. N. Rumanov, Zh. Prikl. Mekh. Tekh. Fiz., No. 1, 7-13 (1985).
3. E. A. Vorob'ev, V. F. Mikhailov, and A. A. Kharitonov, SHF Dielectrics under High-Temperature Conditions [in Russian], Moscow (1977).
4. V. L. Balkevich, Technical Ceramics [in Russian], Moscow (1984).
5. F. Crate and W. Black, Fundamentals of Heat Transfer [Russian translation], Moscow (1983).
6. V. V. Nikol'skii, Electrodynamics and the Propagation of Radio Waves [in Russian], Moscow (1973).
7. Yu. S. Arkhangel'skii and I. I. Devyatkin, Superhigh Frequency Heating Installations for the Intensification of Engineering Processes [in Russian], Saratov (1983).
8. A. A. Samarskii, The Theory of Difference Schemes [in Russian], Moscow (1983).

AN ANALYTICAL MODEL OF THE STRESS–STRAIN STATE OF AN AXISYMMETRIC ELASTIC BODY UNDER CONDITIONS OF A TWO-DIMENSIONAL TEMPERATURE FIELD

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We have derived an analytical solution for the thermoelasticity problem involving the stress–strain state of an axisymmetric body subjected to the action of a two-dimensional temperature field.

We are called upon to deal with the problem of studying the stress–strain state (SSS) of a cylindrical elastic body in the presence of a temperature distribution that is a function of two spatial coordinates and of time, T = T(r, z, t) [1, 2]. This problem is particularly urgent for combustion-engineering processes which take place under markedly nonsteady and nonisothermal conditions [3]. Propagation of the combustion front over the specimens in these processes result in nonuniform thermal effects both in the lateral and longitudinal directions.

If the temperature is a function solely of one coordinate r and the time t, T = T(r, t), we generally make use of the analytical solutions for plane thermoelasticity problems [2, 4, 5]. Let us note that the ability of the models to solve this problem is based on the hypothesis of plane sections. Within the framework of this hypothesis, for an SSS symmetrical relative to the z axis it is possible to determine only the normal stresses, whereas the tangential stresses are assumed to be equal to zero. If the thermal effects are nonuniform along the length of the cylinder, the plane sections undergo bending. In this case, the tangential stresses may prove to be significant and they cannot be ignored. The problem becomes more complicated if we take into consideration the two-dimensionality of the temperature field and for the solution of the problem we generally make use of numerical methods. At the same time, for a number of questions which require both qualitative and quantitative investigation, it might prove to be useful to have an analytical solution of the three-dimensional problem. Among these questions we can point to the following: determining the criterial

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conditions which make possible the application of certain analytical models from the plane theory of thermoelasticity, and also a study of the role played by tangential stresses in the SSS of the material.

In this study we have found an analytical solution for the thermoelastic problem pertaining to the SSS of a cylindrical body under the conditions of a two-dimensional temperature field, when the strain gradients in the longitudinal direction are small.

1. Let us examine the SSS of a long thin-walled cylinder with an inside radius a_0 and an outside radius b_0, subject to the action of a temperature field. We will assume that external forces are absent on the contour, and that the temperatures producing the stressed state in the material, symmetrically relative to the axis of the cylinder, depends on two coordinates and on time: $T = T(r, z, t)$. We will employ the quasistatic approach [6] in which time enters only as a parameter, the deformation process is steady, and the variations in the mechanical characteristics are subject to a change in the temperature which is assumed to be given. With this approach, the thermal portion of the problem is solved independently of the mechanical portion, which is valid if the temperature field is independent of the strains which it produces.

Let us assume that the deformation pattern changes only slightly in the longitudinal direction, i.e., the distribution of the deformations in altitude must be subject to the following conditions:

$$\frac{\partial \varepsilon_z}{\partial z} = \varepsilon_1 (r,\ z), \quad \frac{\partial \varepsilon_\theta}{\partial z} = \varepsilon_2 (r,\ z), \quad \frac{\partial \varepsilon_r}{\partial z} = \varepsilon_3 (r,\ z), \tag{1}$$

where $\varepsilon_i(r, z)$ are small functions. The order of smallness in ε_i will be determined below. Let us consider the principal differences in the assumption defined by equalities (1) and the plane-section hypothesis used in the analytical model of the "infinite cylinder" [2]. The sections perpendicular to the z axis remain two-dimensional when the following conditions are met:

1) the strains are independent of the z coordinate, i.e.,

$$\frac{\partial \varepsilon_z}{\partial z} = 0, \quad \frac{\partial \varepsilon_\theta}{\partial z} = 0, \quad \frac{\partial \varepsilon_r}{\partial z} = 0\ ; \tag{2}$$

2) all of the points of the fixed section z = const exhibit identical longitudinal displacements w ($\partial w/\partial r = 0$), while the tangential-stress component τ_{rz} is equal to zero (the remaining tangential-stress components $\tau_{z\theta}$ and $\tau_{r\theta}$ are equal to zero because of the conditions of symmetry).

The nonuniformity of the thermal effect over the length of the cylinder leads to a distortion of the shape of the plane sections and to their bending. In this case, the tangential stresses may prove to be significant and they cannot be neglected. Conditions (1) also encompass those cases in which the shearing strain and the tangential stresses are not equal to zero, which makes it possible to study their influence on the SSS of the material. Let us note that the satisfaction of these conditions must be achieved as a consequence of certain limitations on the form of the temperature function $T = T(r, z, t)$.

Let the material be homogeneous and isotropic, while with consideration of the temperature strains the rheological behavior of the material is subject to Hooke's law, which we will write in Lamé form:

$$\begin{gathered}
\sigma_r = \lambda (\varepsilon_r + \varepsilon_\theta + \varepsilon_z) + 2G\varepsilon_r - \frac{\alpha E}{1-2\nu}\Delta T, \\
\sigma_\theta = \lambda (\varepsilon_r + \varepsilon_\theta + \varepsilon_z) + 2G\varepsilon_\theta - \frac{\alpha E}{1-2\nu}\Delta T, \\
\sigma_z = \lambda (\varepsilon_r + \varepsilon_\theta + \varepsilon_z) + 2G\varepsilon_z - \frac{\alpha E}{1-2\nu}\Delta T, \quad \tau_{rz} = G\gamma_{rz}.
\end{gathered} \tag{3}$$

Generally speaking, the moduli of E and G and the coefficients ν and α change over time. The variability of E, G, ν, and α is governed by the nonsteadiness and nonuniform heating of the material at high temperatures. We will assume that all of these parameters are constant, and we will refer these to the mean temperature of the process which is assumed to be known.

Taking the Cauchy equations into consideration, these linking the strain component and the components of the displacements:

$$\varepsilon_r=\frac{\partial u}{\partial r},\quad \varepsilon_\theta=\frac{u}{r},\quad \varepsilon_z=\frac{\partial w}{\partial z},\quad \gamma_{rz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r},\quad \gamma_{r\theta}=\gamma_{z\theta}=0, \tag{4}$$

and substituting (4) into (3), we obtain an expression for Hooke's law in terms of the displacements:

$$\begin{aligned}
\sigma_r&=\lambda\left(\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}\right)+2G\frac{\partial u}{\partial r}-\frac{\alpha E}{1-2\nu}\Delta T,\\
\sigma_\theta&=\lambda\left(\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}\right)+2G\frac{u}{r}-\frac{\alpha E}{1-2\nu}\Delta T,\\
\sigma_z&=\lambda\left(\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}\right)+2G\frac{\partial w}{\partial z}-\frac{\alpha E}{1-2\nu}\Delta T,\\
\tau_{rz}&=G\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)
\end{aligned} \tag{5}$$

(because of the symmetry conditions no tangential displacement occurs: v = 0).

The equilibrium equations written for the case in which there are no external forces have the following form:

$$\frac{\partial \sigma_r}{\partial r}+\frac{\partial \tau_{rz}}{\partial z}+\frac{\sigma_r-\sigma_\theta}{r}=0,\quad \frac{\partial \tau_{rz}}{\partial r}+\frac{\partial \sigma_z}{\partial z}+\frac{\tau_{rz}}{r}=0. \tag{6}$$

Substitution of expressions (5) into (6) gives us the equilibrium equations in the displacements:

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}=g_1\frac{\partial T}{\partial r}-g_3\frac{\partial^2 w}{\partial r\partial z}-g_4\frac{\partial^2 u}{\partial z^2}, \tag{7}$$

$$\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}=g_2\frac{\partial T}{\partial z}-g_5\left(\frac{\partial^2 u}{\partial r\partial z}+\frac{1}{r}\frac{\partial u}{\partial z}\right)-\frac{1}{g_4}\frac{\partial^2 w}{\partial z^2}. \tag{8}$$

Taking the Cauchy equations into consideration in (1) for displacements (4), we obtain

$$\begin{aligned}
&\frac{\partial \varepsilon_z}{\partial z}=\frac{\partial^2 w}{\partial z^2}=\varepsilon_1(r,z),\quad \frac{\partial \varepsilon_\theta}{\partial z}=\frac{1}{r}\frac{\partial u}{\partial z}=\varepsilon_2(r,z),\\
&\frac{\partial \varepsilon_r}{\partial z}=\frac{\partial^2 u}{\partial r\partial z}=\varepsilon_3(r,z).
\end{aligned} \tag{9}$$

Let us determine the order of smallness for ε_i in (9). We will assume that the terms ε_i in Eq. (8) exhibit the following order of smallness:

$$|\varepsilon_i|\ll\left|g_2\frac{\partial T}{\partial z}\right|. \tag{10}$$

In addition, we will assume the condition that the second derivative in Eq. (7) is small:

$$\frac{\partial^2 \varepsilon_\theta}{\partial z^2}=\frac{1}{r}\frac{\partial^2 u}{\partial z^2}\ll g_1\frac{\partial T}{\partial r}. \tag{11}$$

Neglecting the small terms from (10) and (11) in Eqs. (7) and (8), we obtain:

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}=g_1\frac{\partial T}{\partial r}-g_3\frac{\partial^2 w}{\partial r\partial z}, \tag{12}$$

$$\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}=g_2\frac{\partial T}{\partial z}. \tag{13}$$

Having twice integrated Eqs. (12) and (13), we find the expressions for u and w:

$$u(r,\ z)=\frac{g_1}{r}\int_{a_0}^{r}T(r,\ z)\,rdr-\frac{g_3}{r}\int_{a_0}^{r}\frac{\partial w}{\partial z}\,rdr+\frac{r^2-a_0^2}{2r}\,a(z)+\frac{b(z)}{r},\tag{14}$$

$$w(r,\ z)=g_2\int_{a_0}^{r}\frac{1}{r}\int_{a_0}^{r}\frac{\partial T}{\partial z}\,rdrdr+\ln\frac{r}{a_0}\,c(z)+d(z).\tag{15}$$

The unknown functions $a(z)$, b(z), c(z), and d(z) which appear as a result of the integration are found from the boundary conditions prevailing at the contour of the cylinder.

Having substited the expressions for u and w into (5), we obtain the formulas for the stresses:

$$\sigma_r=-\frac{2Gg_1}{r^2}\int_{a_0}^{r}Trdr+\frac{2Gg_3}{r^2}\int_{a_0}^{r}\frac{\partial w}{\partial z}\,rdr-G\frac{\partial w}{\partial z}+(\lambda+G)\,a(z)+G\frac{a_0^2}{r^2}\,a(z)-\frac{2Gb(z)}{r^2},$$

$$\sigma_\theta=\frac{2Gg_1}{r^2}\int_{a_0}^{r}Trdr-2Gg_1T(r,\ z)+\lambda g_4\frac{\partial w}{\partial z}-\frac{2Gg_3}{r^2}\int_{a_0}^{r}\frac{\partial w}{\partial z}\,rdr+(\lambda+G)\,a(z)-G\frac{a_0^2}{r^2}\,a(z)+\frac{2Gb(z)}{r^2},\tag{16}$$

$$\sigma_z=(\lambda g_4+2G)\frac{\partial w}{\partial z}-2Gg_1T(r,\ z)+\lambda a(z),$$

$$\tau_{rz}=\frac{G(g_1+g_2)}{r}\int_{a_0}^{r}\frac{\partial T}{\partial z}\,rdr+G\,\frac{r^2-a_0^2}{2r}\,a(z).$$

In comparison with the calculation of stresses based on the original system of differential equations (7) and (8), the calculations based on formulas (16) is considerably simpler, although utilization of the found analytical solution is based on certain simplifying assumptions [see (1)].

2. The possibility of realizing conditions (1) is associated with certain limitations imposed on the nature of the external effects, in this particular case on the temperature function. It is physically obvious that in this case the temperature gradient along the length cannot be arbitrary. Let us find these limitations.

Essentially, relationships (1) can be regarded as differential equations for the determination of the displacements u and w; however, in this case they do not necessarily satisfy the original equilibrium equations (7) and (8). In its complete form this problem can therefore be formulated as follows: find the displacements u and w to satisfy the equilibrium equations (7) and (8), as well as Eqs. (1). Since the number of equations in this case is larger than the number of unknowns, at first glance such a problem appears to be indeterminate. However, if Eqs. (1) are regarded as auxiliary conditions of compatability which must be satisfied by determining the form of the temperature function, the system of equations (1), (7), (8) under consideration is closed.

It must be assumed in the more general case that in (1) $\varepsilon_i=\varepsilon_i(r,\ z)$. The consideration is somewhat simplified if we assume that ε_i = const, which encompasses also the case $\varepsilon_i=0$. Having substituted the Cauchy equations for the displacements into (1), we obtain

$$\frac{\partial^2 w}{\partial z^2}=\varepsilon_1,\quad \frac{1}{r}\,\frac{\partial u}{\partial z}=\varepsilon_2,\quad \frac{\partial^2 u}{\partial r\partial z}=\varepsilon_3.\tag{17}$$

It is obvious that only two of these three equations are independent, since it can be demonstrated that $\varepsilon_2=\varepsilon_3$. Integration of (17) shows that u and w must be represented in the form

$$w(z,\ z)=\varepsilon_1\frac{z^2}{2}+a_1(r)\,z+b_1(r),\tag{18}$$

$$u(r,\ z)=\varepsilon_2 rz+a_2(r),\tag{19}$$

where $a_1(r)=\partial w/\partial z(r,\ 0)$; $b_1(r)=w(r,\ 0)$; $a_2(r)=u(r,\ 0)$. Having substituted these expressions into Eqs. (7) and (8), we have

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(a_2 r)}{\partial r}\right)=g_1\frac{\partial T}{\partial r}-g_3\frac{\partial a_1}{\partial r}, \tag{20}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial a_1}{\partial r}\right)+\frac{\partial}{\partial r}\left(r\frac{\partial b_1}{\partial r}\right)=g_2\frac{\partial T}{\partial z}r. \tag{21}$$

These equations determine the form of the temperature function T(r, z). Integration of (21) over z gives us

$$T(r,\ z)=f_1(r)\frac{z^2}{2}+f_2(r)z+\varphi(r), \tag{22}$$

where $f_1(r) = (1/r)(\partial/\partial r)[r(\partial a_1/\partial r)]$; $f_2(r) = (1/r)(\partial/\partial r)[r(\partial b_1/\partial r)]$; $\varphi(r) = T(r, 0)$. Since (20) and (21) must define one and the same function T(r, z), the next derivatives $\partial^2 T/(\partial r\partial z)$, determined from (20) and (21), must be equal to each other. Since $\partial^2 T/(\partial r\partial z) = 0$ from (20), then $\partial^2 T/(\partial r\partial z)$ determined from (21) must also be equal to zero, i.e.,

$$\frac{\partial^2 T}{\partial r\partial z}=f_1'(r)z+f_2'(r)=0.$$

Finally, we find that in order to satisfy conditions (17) the temperature function must have the following form:

$$T(r,\ z)=C_1 z^2+C_2 z+\varphi(r), \tag{23}$$

where C_1 and C_2 are arbitrary constants, and $\varphi(r)$ is an arbitrary function.

In the general case in which $\varepsilon_i = \varepsilon_i(r, z)$ in (1), it is no longer possible to determine the form of the temperature function which governs the ability of the model conditions to be satisfied. However, in this case we can indicate the limitations imposed on any change in temperature in the z direction. Let us estimate the difference $|T(r, z) - T(r, 0)|$. Integrating (17) and taking into consideration $\varepsilon_i = \varepsilon_i(r, z)$, we obtain

$$w(r,\ z)=\int_0^z dz\int_0^z \varepsilon_1(r,\ z)\,dz+a_1(r)z+b_1(r), \tag{24}$$

$$u(r,\ z)=r\int_0^z \varepsilon_2(r,\ z)\,dz+a_2(r). \tag{25}$$

Substituting (24) and (25) into (12) and (13):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial a_1}{\partial r}\right)z+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial b_1}{\partial r}\right)+\frac{1}{r}\int_0^z dz\int_0^z\frac{\partial}{\partial r}\left(r\frac{\partial\varepsilon_1}{\partial r}\right)dz=g_2\frac{\partial T}{\partial z}, \tag{26}$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(a_2 r)}{\partial r}\right)+\int_0^z\left(3\frac{\partial\varepsilon_2}{\partial r}+r\frac{\partial^2\varepsilon_2}{\partial r^2}+g_3\frac{\partial\varepsilon_1}{\partial r}\right)dz\quad+g_3\frac{\partial b_1}{\partial r}=g_1\frac{\partial T}{\partial r}. \tag{27}$$

Having twice differentiated (26) with respect to z, we have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varepsilon_1}{\partial r}\right)=g_2\frac{\partial^3 T}{\partial z^3}. \tag{28}$$

Because of the monotonicity of T(r, z) in the bounded region there exist such constants M_1, M_2, and M_3 that

$$\left|\frac{\partial^3 T}{\partial z^3}\right|\leqslant 6M_1,\quad \left|\frac{\partial^2 T}{\partial z^2}\right|\leqslant M_2,\quad \left|\frac{\partial T}{\partial z}\right|\leqslant M_3. \tag{29}$$

From the physical standpoint the following quantities have been limited:

$$\left|\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial a_1}{\partial r}\right)\right|\leqslant 2M_2,\quad \left|\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial b_1}{\partial r}\right)\right|\leqslant M_3. \tag{30}$$

Integrating (26) and taking into consideration (28)-(30), we obtain

$$|T(r,\ z)-T(r,\ 0)|\leqslant M_1z^3+M_2z^2+M_3z. \tag{31}$$

Inequality (31) defines the upper bound of the variations in temperature $T_*(r, z)$ at which the assumptions of the model are satisfied:

$$T_*(r,\ z)=M_1z^3+M_2z^2+M_3z+\varphi(r),\quad |T(r,\ z)|\leqslant|T_*(r,\ z)|. \tag{32}$$

Let us note that as $M_1 \to 0$, limitation (32), obtained at the temperature for the case in which $\varepsilon_i = \varepsilon_i(r, z)$, changes into condition (23) for ε_i = const. In view of (13) and (29), we obtain

$$\left|\frac{\partial^2 w}{\partial z^2}\right|\leqslant CM_1g_2,$$

and according to (9):

$$|M_1|\ll\left|\frac{\partial T}{\partial z}\right|\leqslant M_3.$$

Hence it follows that the coefficient M_1 is small and conditions (32) and (23) are quite close to each other. As was demonstrated earlier, on satisfaction of condition (23), Eq. (27) is also satisfied.

Thus, if the experiment gives us $T(r, z, t)$ (this information can be obtained on the basis of the thermal model by numerical calculation [7]), we are still confronted with the problem of best approximating a temperature function of the form of (23) or (32). If such a problem can be resolved, the assumptions of the model with regard to the limited strain gradients in altitude are satisfied and we can make use of the proffered analytical solutions of (16).

A number of questions pertaining to the derived analytical solution remain beyond the scope of this paper. We have already made mention of the need to find integration constants for specific boundary conditions, of the need to carry out numerical calculations to analyze the SSS of the material, and in particular, to study the effect of tangential stress on the material, as well as to find the criterial conditions for the applicability of known analytical solutions for plane problems from the theory of thermoelasticity. These problems will be dealt with in the following portion of this project.

NOTATION

T, temperature of the body; r, z, θ, the radial, vertical, and tangential cylindrical coordinates; t, time; ε_r, ε_θ, ε_z, deformations in the radial, tangential, and longitudinal directions; γ_{rz}, $\gamma_{r\theta}$, $\gamma_{z\theta}$, shearing strains; σ_r, σ_θ, σ_z, components of the normal stress in the radial, tangential, and longitudinal directions; τ_{rz}, $\tau_{r\theta}$, $\tau_{z\theta}$, tangential stress components; ε_1, ε_2, ε_3, small quantities; λ, G, elastic Lamé constants; ν, Poisson coefficient; α, coefficient of linear temperature expansion; E, Young's modulus; ΔT, elevation of temperature, i.e., the difference between the initial temperature T_0 and the temperature at a given instant of time T(t); u, w, v, displacement vector components in the radial, vertical, and tangential directions; g_1, g_2, g_3, g_4, g_5, constants determined by the following expressions: $g_1 = \alpha(1 + \nu)/(1 - \nu)$, $g_2 = 2\alpha(1 + \nu)/(1 - 2\nu)$, $g_3 = 1/2(1 - \nu)$, $g_4 = (1 - 2\nu)/2(1 - \nu)$, $g_5 = 1/(1 - 2\nu)$; M_1, M_2, M_3, C_1, C_2, C are constants.

LITERATURE CITED

1. M. F. Mikhalev (ed.), Design and Construction of Machines and Equipment in the Chemical Industry [in Russian], Leningrad (1973).
2. A. N. Podgornyi, G. A. Marchenko, and V. I. Pustynnikov, Fundamentals and Methods in the Applied Theory of Elasticity [in Russian], Kiev (1981).
3. A. G. Merzhanov, "The combustion process: theory and practice," Preprint, OIKhF AN SSSR, Chernogolovka (1980).
4. S. P. Timoshenko, The Theory of Elasticity [Russian translation], Moscow (1937).
5. L. I. Sedov, The Mechanics of Continous Media [in Russian], Vol. 2, Moscow (1970).
6. G. Parkus, Nonsteady Temperatures Stresses [Russian translation], Moscow (1963).
7. L. S. Stel'makh and A. M. Stolin, Theory of Mechanical Processing of Polymer Materials. Abstracts of Reports to the All-Union Symposium, Perm' (1985), p. 173.